DESCRIPTION OF THE REDUCTION IN TURBULENT FRICTION DRAG IN VISCOELASTIC FLUIDS

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A model description of the near-wall turbulent fluid flow with stress relaxation and aftereffect is proposed. It is shown that a large reduction in the friction drag and heat transfer occurs.

Much literature (see the survey [1]) has been devoted to experimental investigations of turbulent viscous fluid flows to which have been added macromolecular compounds, long fibers, etc. Among the diverse viewpoints on the reasons for these phenomena, the hypothesis about the governing role of the elastic properties of such solutions has gained great popularity, starting with the early experimental researches [2-4].

It is known that upon the addition of polymers to a bounded turbulent stream, the turbulence characteristics near the walls undergo a fundamental change while they vary insignificantly far from them. Investigations of viscous fluid turbulence [5-14] also indicate the leading role of the near-wall zone. The smooth laminar flow with quasi-periodic spatial characteristics being formed here is subjected to abrupt distortions ("explosions") sufficiently regularly. Chaotic turbulent pulsations also originate during these "explosions." A schematic description of such a self-oscillating process of a viscous fluid flow (in the "viscous sublayer" domain) had already been proposed comparatively long ago [5, 15, 16]. An analogous schematization is used here to describe the flow of a viscoelastic fluid.†

External Turbulence. In describing steady developed turbulent flow, its whole domain can be separated into two zones rather conditionally. The external domain, far from the wall, has a large scale structure independent of the specific fluid properties (viscosity, elasticity, etc.). A theoretical description of this zone is difficult because of the strong nonlinear interaction between the turbulent motions of the various scales. However, at distances also far from the outer edge of the stream (from the center of the pipe) the total mean shear stress becomes constant and equal to the stress at the wall, and the form of the mean velocity profile is determined by dimensionality considerations

$$2.5 \ln z^{+} + B. \tag{1}$$

The fluid is henceforth considered incompressible, and the system of measurement units is selected such that the density is $\rho = 1$. The dynamic velocity u_* and the viscosity coefficient ν hence form a complete set of quantities with independent dimensionality and it is convenient to make all the quantities dimensionless with their aid. The dimensionless characteristics obtained in such a way are ordinarily marked with a cross (+) over the appropriate dimensional letter value. Henceforth, mainly such dimensionless characteristics are used and the crosses are omitted to simplify the writing.

The numerical coefficient 2.5 is known from experiment. The coefficient B can depend on the fluid properties, and takes the following value for a viscous fluid

$$B_0 \approx 5.5. \tag{2}$$

†To the same end, a model description of the type [5, 15, 16] has already been considered earlier in [17-20]. However, no sufficiently complete quantitative analysis of the drag reduction has been obtained in these papers.

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Dynamic Layer. The rheological properties of the fluid play the governing role in direct proximity to the wall. For a large part of the time the flow here is nonstationary and laminar, where the nonlinear peculiarities in the fluid behavior are important principally in the fast stage of destruction of this smooth flow (in the "explosions"). Assuming the fast stage of the "explosions" not to yield a noticeable contribution to such mean characteristics as the mean velocity and the mean rheological stress (see [5]), let us henceforth use simplified linear equations for their computation.

Many parameters, the frequency of the explosions, say, remain undetermined in the solution of the linear problem. Additional "splicing" conditions for the mean characteristics in this dynamical zone and in the domain of external turbulence permit calculation of their magnitude in such a description.

Let us describe the flow of a viscoelastic fluid in the near-wall dynamical layer by one-dimensional linear equations

$$\frac{\partial u}{\partial t} = \frac{\partial \sigma}{\partial z} + P, \quad \left(1 + \theta \frac{\partial}{\partial t}\right)\sigma = \left(1 + \gamma\theta \frac{\partial}{\partial t}\right)\frac{\partial u}{\partial z}. \tag{3}$$

The pressure gradient P is considered constant here. For an elastic fluid

$$\theta > 0; \qquad 1 > \gamma \geqslant 0, \tag{4}$$

where the "elastic stress" is proportional to $1-\gamma$ and $\gamma\theta$ is the time of the after effect [21].

The simplified equations (3) can be used to describe the smooth stage of flow development, but not at the time of the "explosions." Let us consider that the system again arrives in the initial state each time as a result of the instantaneous "explosions" occurring at random times.

Then if the "explosions" occur independently, the stationary random processes u(t, z) and σ(t, z) will be Poisson processes with mean characteristics of the form

$$\langle f(z) \rangle = \frac{1}{T}\int_0^\infty \exp(-t/T) f(z, t)\, dt, \tag{5}$$

i.e., the mean characteristics of the dynamical layer are obtained by a Laplace transformation of the instantaneous values.

After each "explosion" the turbulent mixing process includes the whole flow domain down to the wall. Hence, constant values of the velocity and the tangential stress can be given as the simplest initial conditions for (3). Together with the boundary conditions they will be

$$u(z, 0) = U_0, \quad \sigma(z, 0) = \Sigma, \quad u(0, t) = 0, \quad |u| < \infty. \tag{6}$$

The appearance of a nonzero initial stress is due to the elastic properties of the fluid, and since the elastic stresses are proportional to $1-\gamma$ in the case under consideration, then

$$\Sigma = \delta(1-\gamma) \tag{7}$$

with a number δ on the order of one.

The solution of the system (3), (4), (6) results in the formulas

$$\langle u(z) \rangle = U(1 - \exp(-\mu z)), \tag{8}$$

$$\langle \sigma(z) \rangle = \frac{\theta}{T+\theta}\Sigma + \frac{U}{\mu T}\exp(-\mu z), \tag{9}$$

$$U = U_0 + PT, \quad \mu T^{1/2} = \left(\frac{T+\theta}{T+\gamma\theta}\right)^{1/2}. \tag{10}$$

Furthermore, from the assumption about the constancy of the total stress in the dynamical layer (this is characteristic for developed turbulence when the thickness of the dynamical layer is very much less than the pipe radius), there follows that the sum of the rheological stress $\langle\sigma\rangle$ and the turbulent Reynolds shear stress $\langle u'v'\rangle$ equals the stress on the wall, i.e.,

$$\langle \sigma(z) \rangle - \langle u'v' \rangle = 1. \tag{11}$$

The Reynolds stress vanishes for z = 0 and taking account of (9), we have the following relationship from (11)

$$U/\mu T + \theta\Sigma/(T+\theta) = 1. \tag{12}$$

Splicing Conditions. Other relationships between these parameters are obtained from the conditions of splicing the dynamical layer and the external turbulence distributions

$$z_1 \cdot \left\langle \frac{\partial u}{\partial z} \right\rangle \Big|_{z=0} = 2.5 \ln z_1 + B, \quad \langle \sigma(z_1) \rangle = \alpha_1; \tag{13}$$

$$\langle u(z_2) \rangle = 2.5 \ln z_2 + B, \quad \langle \sigma(z_2) \rangle = \alpha_2.$$

The first point z_1 (the point of intersection between the linear asymptote valid in direct proximity to the wall and the logarithmic profile of the external turblence (1)) is also characteristic by the fact that the rheological stress (viscous friction in a viscous fluid) at this point is approximately equal in magnitude to the Reynolds shear stress. In conformity with (11), the assumption about their exact equality reduces to the second formula in (13) with

$$\alpha_1 = 1/2. \tag{14}$$

Hence, at the point z_2 where the velocities of the dynamical layer (8) and the external turbulence (1) are equal, the viscous stress also turns out to be constant in the case of a viscous fluid, i.e., the fourth relationship in (13) is satisfied automatically and

$$\alpha_2 \approx 0.164\ldots \tag{15}$$

Since the point z_2 is the exact outer boundary of the dynamical layer, it can then be expected that this condition will be independent of the specific properties of the fluid, just as the other properties of the external turbulence. On this basis (14) is assumed valid even for a viscoelastic fluid.

In sum, using (8), (9), (13)-(15), we obtain

$$z_k = \mu^{-1}\{\ln[T + \theta - \delta(1-\gamma)\theta] - \ln[(T+\theta)\alpha_k - \delta(1-\gamma)\theta]\}, \tag{16}$$

$$U = 2.5 \ln(z_2/z_1)[1 - \mu z_1 - \exp(-\mu z_2)]^{-1}, \tag{17}$$

$$B = \mu U z_1 - 2.5 \ln z_1 \quad (k = 1, 2). \tag{18}$$

The relationships (7), (12), (16), (17) permit the determination of T as a function of θ and γ. In turn, for a known T formula (18) permits finding B and thereby the complete determination of the distribution (1). Integrating this distribution over the cross section of a circular pipe, we obtain a formula for the turbulent friction drag† (q is the mean discharge velocity of the fluid)

$$\sqrt{8/\lambda} = 2.5 \ln(\mathrm{Re}\sqrt{\lambda/32}) + B - 3.75, \tag{19}$$

$$\mathrm{Re} = 2qr, \quad \lambda \equiv 8/q^2. \tag{20}$$

In the case of a viscous fluid the coefficient B is here given by (2) and in the case of a viscoelastic fluid by (18). The quantity B in terms of the dimensionless parameter θ hence turns out to be dependent on the dynamic velocity u_*. In order to go over to a dependence of B on Re and λ, it is convenient to introduce the new dimensionless characteristic

$$\tilde{r} = r/(2\sqrt{\theta}); \quad \sqrt{\theta} = (\mathrm{Re}/(2\tilde{r}))\sqrt{\lambda/32}. \tag{21}$$

The dimensionless characteristic $\tilde{r}$ is independent of the quantity u_* and is expressed by $\tilde{r} = r/(2\sqrt{\nu\theta})$ in terms of the dimensional characteristics.

The flow will differ slightly from the viscous when the dimensionless parameter θ is small. Hence $T/\theta \gg 1$ and the preceding formulas can be written in the form of the expansions

$$z_1 \approx 11.6 + 0.14\delta(1-\gamma)\theta + \ldots, \tag{22}$$

$$z_2 \approx 30.3 + 0.52\delta(1-\gamma)\theta + \ldots,$$

$$U \approx \sqrt{T} \approx 16.8 + 0.15\delta(1-\gamma)\theta + \ldots,$$

$$B \approx 5.5 + 0.13\delta(1-\gamma)\theta + \ldots$$

Within the framework of the same accuracy, we have for the drag coefficient

$$\lambda/\lambda_0 \approx 1 - 0.0007\delta(1-\gamma)\lambda_0^{3/2}(\mathrm{Re}/\tilde{r})^2, \tag{23}$$

where λ_0 is the drag coefficient of a viscous fluid.

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†For simplicity, the profile (1) is considered to hold down to the pipe axis although, strictly speaking, it is not true there. Just the same as neglecting details of the velocity distribution in the dynamical layer for $z_2 \ll r$, this does not introduce a large error in (19).

TABLE 1. Limit Values of the Parameters z_1, z_2, T and B in a Viscoelastic Fluid with Large Aftereffect

γ	0,85	0,88	0,91	0,94	0,97	1,0
$\bar{z}_1$	35,5	24,2	18,8	15,4	13,3	11,6
$\bar{z}_2$	163	86,6	59,3	44,6	36,2	30,3
$\bar{T}$	1870	950	615	440	345	282
$\bar{B}$	26,5	16,3	11,5	8,5	6,8	5,5

It is seen from the formulas presented that in the presence of elasticity and for $\delta \neq 0$ the resistance to the fluid flow is reduced, the thickness z_1 of the "viscous sublayer," the thickness z_2 of the "transition layer," the period T of dynamical layer pulsation (the time between "explosions") and the coefficient B of the velocity profile increase.

In the opposite case of small T/θ there are three distinct possibilities. As T/θ decreases, the quantity z_2 grows or can become infinite for some minimum value $(T/\theta)_{min} = P/\alpha_2 > 0$ or reaches a finite maximum value for $T/\theta = 0$ or grows without limit as $T/\theta \to 0$.

In the first case the asymptotically simplified formulas are for $\delta = 1$

$$z_1 \approx B \approx 2.1\sqrt{P\theta}, \quad U = 1.1\sqrt{T} \approx 2.3\sqrt{P\theta}, \quad \lambda = 15\bar{r}/\mathrm{Re}\sqrt{P}, \tag{24}$$

where $P = 1 - \alpha_2 - \gamma > 0$ here.

If $P = 1 - \alpha_2 - \gamma < 0$, then both z_2 and the other quantities reach finite limit values for $T/\theta = 0$

$$\bar{z}_1 = U\ln(1 - 1/(2\gamma))^{-1}, \quad \bar{z}_2 = U\ln(-P/\gamma)^{-1},$$
$$U = 2.5\{(1-\alpha_2)/\gamma + \ln(1 - 1/(2\gamma))\}\ln[\ln(-P/\gamma)/\ln(1-1/(2\gamma))], \tag{25}$$
$$\bar{T} = U^2/\gamma, \quad \bar{B} = \bar{z}_1 - 2.5\ln\bar{z}_1.$$

Numerical values of these characteristics are presented in the table for certain γ. Since B reaches the limit value $\bar{B}$ here, then the limit dependence of λ on Re will be analogous to the dependence in a viscous fluid but with constant $\bar{B}$ in place of B_0. The form of this asymptote is evidently independent of the pipe radius in contrast to what holds in (24).

Finally, if $T/\theta = 0$ is a singularity of the dependence $z_2 = z_2(T/\theta)$, then an intermediate type of behavior holds (curve 5 in Fig. 1 and the dashed curves in Fig. 2 correspond to this case).

In order to determine the behavior of the characteristics under consideration for intermediate values of the parameters θ and T/θ, numerical computations were performed using (8), (9), (16)-(21). The results of these calculations are presented as graphs in Figs. 1-4 for $\delta = 1$.

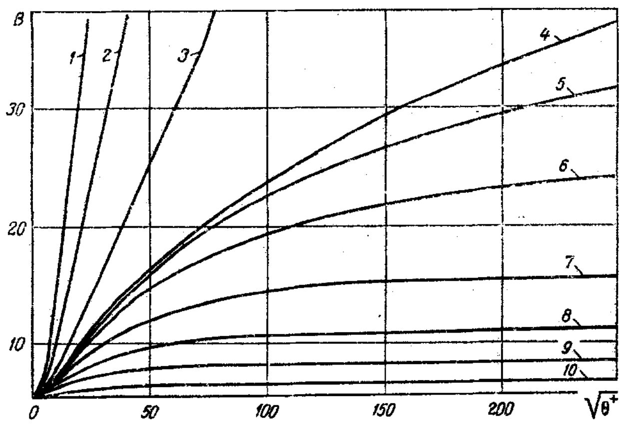

Fig. 1. Dependence of the numerical parameter B on $\sqrt{\theta^+}$ for the values γ = 0 (1); 0.5 (2); 0.75 (3); 0.83 (4); 0.8358... (5); 0.85 (6); 0.88 (7); 0.91 (8); 0.94 (9); 0.97 (10).

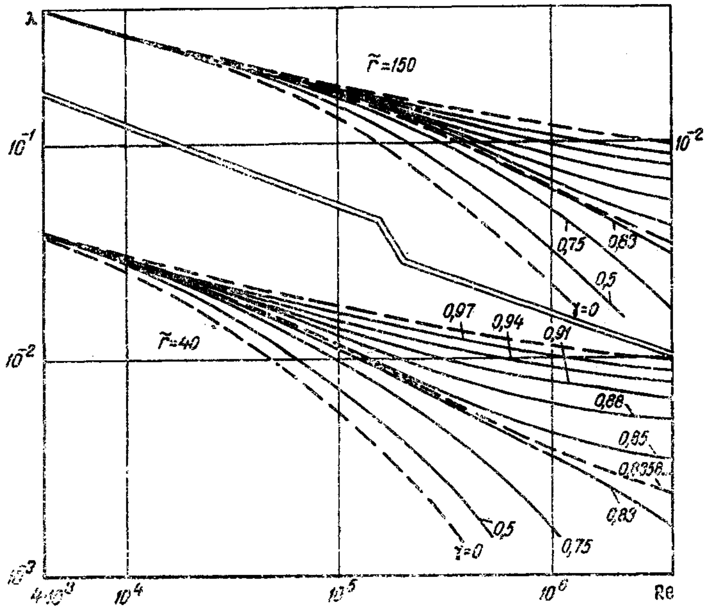

Fig. 2. Curves of the drag $\lambda(\mathrm{Re}, \tilde{r}, \gamma)$ for $\tilde{r}$ equal to 150 and 40, and $\gamma = 0$; 0.5; 0.75; 0.83; 0.8358...; 0.85; 0.88; 0.91; 0.94; 0.97. The drag curves of turbulent friction for a viscous fluid and for a viscoelastic fluid with γ 0.8358... and γ = 0 are drawn dashed.

DISCUSSION OF RESULTS

The behavior of B is illustrated in Fig. 1. It should be noted that it is impossible to attribute great value to the asymptotic behavior of both B and the other characteristics for $\theta \gg 1$ since for high elasticity the simplified description under consideration by using the linear equations (3) is too rough.

It is seen from the drag curves in Fig. 2 that the critical value of the Reynolds number corresponding to the beginning of the drag reduction effect which increases as the pipe radius increases, depends on the magnitude of the parameter θ. The slope of the reduced drag curve relative to the viscous fluid drag curve depends on the magnitude of the parameter γ.

The change in the mean velocity profile in a viscoelastic fluid is shown in Fig. 3. The profile with $T/2\theta = 500$ is close to the profile of a viscous fluid independently of the quantity γ. As $T/2\theta$ decreases

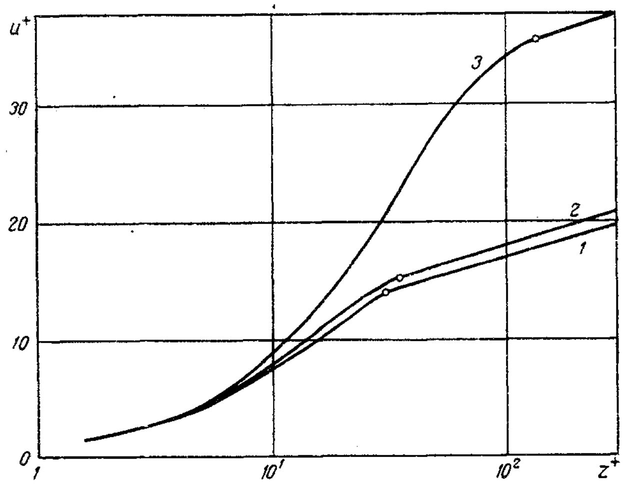

Fig. 3. Mean velocity profiles for $T^+/\theta^+ = 10^3$ (curve 1), $T^+/\theta^+ = 6$ and $\gamma = 0.8358\ldots$ (curve 2), $T^+/\theta^+ = 6$ and $\gamma = 0$ (curve 3). The points correspond to $z^+ = z_2^+$.

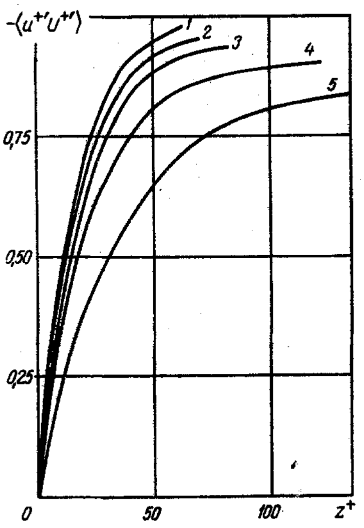

Fig. 4. Dependence of the Reynolds stress $\langle u^{+\prime}v^{+\prime}\rangle$ on the distance to the wall for $\gamma = 0$ and $T^+/\theta^+ = 10^3$ (curve 1), 20 (3), 10 (4), 6 (5) and for $\gamma = 0.8358...$, $T^+/\theta^+ = 6$ (curve 2).

for $\gamma \approx 1$ the profile becomes somewhat more filled. A great difference from the viscous profile holds for small γ and $T/2\theta$.

Finally, Fig. 4 illustrates how the Reynolds stresses change in the constant total stress layer as $T/2\theta$ and γ change. It is hence seen that suppression of the Reynolds stresses occurs as the fluid relaxation time grows.

A comparison between the results obtained and the results of experimental investigations of turbulent flows of polymer solutions [1, 22-24] indicates they have much in common qualitatively. However, a final quantitative comparison is still difficult since the rheological characteristics of these solutions are not known in the majority of cases.

The case $\delta \neq 1$ for $\delta \approx 1$ is not different, in principle, from the case $\delta = 1$. In substance, the distinction reduces to a change in the dependence on γ. However, the situation is different for $\delta = 0$, i.e., when the initial stress Σ is missing.* Hence, (12), (16)-(18) become

$$z_2 = 2.61z_1 = 1.81T/U, \quad U = U_0 = 16.8,$$
$$2T = \{U_0^2 - 0 + \sqrt{(U_0^2 - 0)^2 + 4\gamma 0 U_0^2}\},$$
$$B = B_0 - 2.5\ln(T/U_0^2), \tag{26}$$

from which $T < T_0 = U_0^2$ and $\partial T/\partial\theta < 0$ follow, i.e., the period of near-wall layer oscillation turns out to be less than the period of layer oscillation in a viscous fluid and drops as the elasticity grows. And z_1 and z_2 diminish correspondingly. As regards the drag curves, they then have the shape for $\lambda = 0$, of curves going over from viscous type drag curves to asymptotics of the form

$$\lambda \approx 2(8U\tilde{r}/\mathrm{Re})^2. \tag{27}$$

Such a nature for the behavior of z_1, z_2, T and λ is sharply different from what is observed for polymer solutions.

Heat Transfer. In conclusion, within the framework of the model description of turbulence under consideration, let us discuss other transfer processes. The profile of a passive impurity concentration (temperature, say) in the external domain of a developed turbulent stream $\mathrm{Re} \gg 1$, $\mathrm{Pe} \gg 1$ has the logarithmic form

$$A^c \ln z^+ + B^c \tag{28}$$

exactly as the mean velocity profile (1).

The concentration profile within the oscillating dynamic layer, measured from the concentration at the walls, has a form analogous to (8):

$$\langle c^+ \rangle = \sqrt{\mathrm{Pr}\,T}\,(1 - \exp(-z\sqrt{\mathrm{Pr}/T})). \tag{29}$$

*Namely this case $\Sigma = 0$ was discussed in [18-20].

If the point z_2 is taken as the junction of the distributions (28) and (29), then we obtain the following formula for B^c

$$B^c = \sqrt{\Pr T}\,(1 - \exp(-z_2\sqrt{\Pr/T})) - A^c \ln z_2, \quad (30)$$

which practically reduces to the relationship

$$B^c \approx \sqrt{\Pr T} \quad (31)$$

by virtue of the values of the parameters entering therein.

This same relationship is indeed obtained for a smooth merger of (28) and (29).

The formula for the heat transmission coefficient St becomes

$$\mathrm{St} = \frac{\lambda/8}{0.4A^c + (B^c - 0.4A^c B)\sqrt{\lambda/8}} \sim \sqrt{\frac{\lambda}{8T\Pr}}, \quad (32)$$

from which it is seen that an abrupt reduction in the heat transmission coefficient occurs simultaneously with the diminution in the friction drag. The deductions of experimental investigations of heat transmission in polymer solutions [25] agree qualitatively with the predictions following from (28)-(32).

NOTATION

x	is the longitudinal coordinate;
z	is the distance from the wall;
$z = z_1$	is the viscous sublayer thickness;
$z = z_2$	is the dynamic layer thickness;
t	is the time;
T	is the period of dynamic layer pulsation;
ν	is the kinematic viscosity coefficient;
θ	is the relaxation time;
γ	is the ratio between the aftereffect and relaxation times;
u	is the velocity;
σ	is the stress;
U_0, Σ	are the initial velocity and stress, respectively;
P	is the pressure;
u_*	is the dynamic velocity;
$\langle u'v' \rangle$	is the Reynolds stress;
Re	is the Reynolds number defined by means of the mean flow rate velocity q and the pipe diameter 2r;
Pr	is the Prandtl number;
St	is the Stanton number;
Pe	is the Peclet number;
α_1, α_2, B_0, δ	are the numerical coefficients;

$z^+ = zu_*/\nu$;
$z_i^+ = z_i u_*/\nu$;
$(i = 1, 2)$;
$r^+ = ru_*/\nu$;
$\tilde{r} = r/(2\sqrt{\nu\theta})$;
$t^+ = tu_*^2/\nu$;
$T^+ = Tu_*^2/\nu$;
$u^+ = u/u_*$;
$\sigma^+ = \sigma/u_*^2$;
$\Sigma = \Sigma/u_*^2$;
$U_0^+ = U_0/u_*$;
$P^+ = (dp/dx)\nu/u_*^3$;
$\theta^+ = \theta u_*^2/\nu$ are the corresponding dimensionless values.

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